The matching coefficient of the vector current and the decay $\Upsilon(1S) \to \ell\ell$

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DESY

in collaboration with

J. Piclum, D. Seidel and M. Steinhauser `PRD 89 (2014) 034027`
+ M. Beneke, Y. Kiyo, A. Penin, `PRL 112 (2014) 151801`

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Outline

1. Matching Coefficient of the Vector Current
2. Application: $\Gamma(\Upsilon(1S) \to \ell\ell)$

Outline

Introduction

Physics of bound states of heavy particles and threshold phenomena best described within an effective field theory – Non-Relativistic QCD (NRQCD) and potential Non-Relativistic QCD (pNRQCD)

Prominent applications are

- production of $t\bar{t}$ pairs at threshold at a future linear collider
- decays of $b\bar{b}$ bound states
- $b\bar{b}$ sum rules
- positronium spectra

Matching Procedure

Chain of effective field theories: QCD $\rightarrow$ NRQCD $\rightarrow$ p(otential)NRQCD

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QCD vector current	NRQCD vector current
$j_v^\mu = \bar{Q}\gamma^\mu Q$	$\tilde{j}_v^k = \phi^\dagger \sigma^k \chi$

$$j_v^k = c_v \tilde{j}_v^k + \frac{d_v}{6M^2}\phi^\dagger \sigma^k D^2 \chi + \cdots$$

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$$j_v^k = c_v \tilde{j}_v^k + \mathcal{O}\left(\frac{1}{M^2}\right)$$

c_v can be extracted by calculating vertex corrections involving j_v and $\tilde{j}_v$

$$Z_2 \Gamma_v = c_v \tilde{Z}_2 \tilde{Z}_v^{-1} \tilde{\Gamma}_v + \cdots$$

Details

full and effective theory contain the same soft, ultra-soft and potential contributions $\Rightarrow$ sufficient to calculate vertex functions at threshold

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renormalization of the vector current (effective theory) ✓

Setup of the Calculation

- Feynman diagrams generated using QGRAF [Nogueira]
- mapped onto 78 topologies using Q2E/EXP [Harlander,Seidensticker,Steinhauser]
- Feynman integrals reduced to master integrals with CRUSHER [PM,Seidel]
- master integrals in different topologies have to be identified
- $\mathcal{O}(100)$ master integrals calculated analytically/numerically using various techniques, e.g. sector decomposition implemented in FIESTA [Smirnov]
- numerical errors added in quadrature

Results

$$c_v \approx 1 - 2.667\frac{\alpha_s^{(n_l)}}{\pi} + \left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^2 [-44.551 + 0.407 n_l]$$
$$+ \left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^3 \left[-2091(2) + 120.66(0.01)\, n_l - 0.823\, n_l^2\right]$$
$$+\text{singlet terms}$$

- large NNNLO correction
- but, on its own not a physical quantity
- preliminary results confirm that singlet terms are small

Checks

- Renormalization constant $\tilde{Z}_v$ of the NRQCD current can be reproduced
  - $\tilde{Z}_v$ analytically known, $1/\epsilon$ part numerically small
  - agreement within the error estimate at the percent level

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	default basis	alternative basis
c_{FFF}	$36.55(0.11)$	$36.61(2.93)$
c_{FFA}	$-188.10(0.17)$	$-188.04(2.91)$
c_{FAA}	$-97.81(0.08)$	$-97.76(2.05)$
$c_v^{(3)}\ (n_l = 4)$	$-1621.7(0.4)$	$-1621(23)$
$c_v^{(3)}\ (n_l = 5)$	$-1508.4(0.4)$	$-1507(23)$

Outline

Framework

- Calculated in the framework of pNRQCD
- Master formula

$$\Gamma(\Upsilon(1S) \to \ell^+\ell^-) = \frac{4\pi\alpha^2}{9m_b^2} |\psi_1(0)|^2 c_v \left[c_v - \frac{E_1}{m_b}\left(c_v + \frac{d_v}{3}\right) + \ldots\right]$$

[Beneke,Kiyo,Schuller]

- Wave function ψ_1 and binding energy E_1 calculated in pNRQCD

[Beneke,Kiyo,Penin,Schuller]

- Matching coefficients c_v and d_v as discussed before
- First test of perturbative bound-state dynamics where all scales (hard, soft, ultrasoft) are present

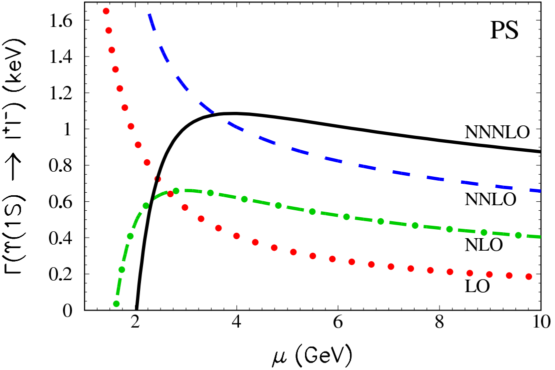
Perturbative Corrections – Pole scheme

$$
\begin{aligned}
&\Gamma(\Upsilon(1S) \to \ell^+\ell^-)|_{\text{pole}} \\
&= \frac{2^5\alpha^2\alpha_s^3 m_b}{3^5}\Big[1 + \alpha_s\,(-2.003 + 3.979\,L) \\
&\quad + \alpha_s^2\left(9.05 - 7.44\,\ln\alpha_s - 13.95\,L + 10.55\,L^2\right) \\
&\quad + \alpha_s^3\,(-0.91 + 4.78_{a_3} + 22.07_{b_{2\epsilon}} + 30.22_{c_f} \\
&\qquad - 134.8(1)_{c_g} - 14.33\,\ln\alpha_s - 17.36\,\ln^2\alpha_s \\
&\qquad + (62.08 - 49.32\,\ln\alpha_s)\,L - 55.08\,L^2 + 23.33\,L^3\Big) + \mathcal{O}(\alpha_s^4)\Big] \\
&= \frac{2^5\alpha^2\alpha_s^3 m_b}{3^5}\Big[1 + 1.166\alpha_s + 15.2\alpha_s^2 + (66.5 + 4.8_{a_3} \\
&\qquad + 22.1_{b_{2\epsilon}} + 30.2_{c_f} - 134.8(1)_{c_g})\,\alpha_s^3 + \mathcal{O}(\alpha_s^4)\Big] \\
&= \frac{2^5\alpha^2\alpha_s^3 m_b}{3^5}\,[1 + 0.28 + 0.88 - 0.16] = [1.04 \pm 0.04(\alpha_s)^{+0.02}_{-0.15}(\mu)]\ \text{keV}
\end{aligned}
$$

Perturbative corrections – PS scheme

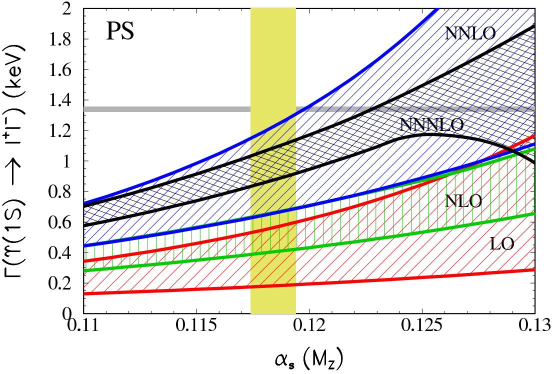
$$
\begin{aligned}
&\Gamma(\Upsilon(1S) \to \ell^+\ell^-)|_{\rm PS} \\
&= \Gamma(\Upsilon(1S) \to \ell^+\ell^-)|_{{\rm pole}, m_b \to m_b^{\rm PS}} \\
&\quad + \frac{2^5\alpha^2\alpha_s^3 m_b^{\rm PS}}{3^5}\frac{\mu_f}{m_b^{\rm PS}\alpha_s}\Big[0.42\alpha_s^2 + \alpha_s^3\left(-1.78 + 0.28\,L_f + 1.69\,L\right) + \mathcal{O}(\alpha_s^4)\Big] \\
&= \frac{2^5\alpha^2\alpha_s^3 m_b^{\rm PS}}{3^5}\Big[1 + 1.528\alpha_s + 16.3\alpha_s^2 + (74.7 + 4.8_{a_3} \\
&\qquad\qquad +22.1_{b_{2\epsilon}} + 30.2_{c_f} - 134.8(1)_{c_g})\,\alpha_s^3 + \mathcal{O}(\alpha_s^4)\Big] \\
&= \frac{2^5\alpha^2\alpha_s^3 m_b^{\rm PS}}{3^5}\left[1 + 0.37 + 0.95 - 0.04\right] = \left[1.08 \pm 0.05(\alpha_s)^{+0.01}_{-0.20}(\mu)\right]\ \text{keV}
\end{aligned}
$$

Results – μ dependence



- NNNLO contribution of moderate size
- improved scale dependence
- no apparent convergence below $\mu \lessapprox 3\,\mathrm{GeV}$
- choose $\mu \in [3, 10]\,\mathrm{GeV}$

Results – α_s dependence



- $\Gamma(\Upsilon(1S) \to \ell^+\ell^-)_{\mathrm{PS}} = [1.08(5)^{+0.01}_{-0.20}]$ keV $\mu \in [3, 10]$ GeV
- $\Gamma(\Upsilon(1S) \to \ell^+\ell^-)_{\mathrm{exp}} =$ 1.340(18) keV
- theory prediction well below exp. value
- sizeable non-perturbative contribution?

Non-perturbative contribution

- Non-perturbative contribution from gluon condensate

$$\delta_{\mathrm{np}}|\psi_1(0)|^2 = |\psi_1^{\mathrm{LO}}(0)|^2 \times 17.54\pi^2 K\,, \qquad K = \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{m_b^4 (\alpha_s C_F)^6}$$

[Pineda; Voloshin]

With $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012\ \mathrm{GeV}^4$ and $\alpha_s(3.5\,\mathrm{GeV})$
$\Rightarrow \delta_{\mathrm{np}}\Gamma_{\ell\ell}(\Upsilon(1S))_{\mathrm{pole}} = 1.67$ keV and $\delta_{\mathrm{np}}\Gamma_{\ell\ell}(\Upsilon(1S))_{\mathrm{PS}} = 2.20$ keV

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- Obtain K by comparing with mass extraction

$$M_{\Upsilon(1S)} = 2m_b + E_1^{\rm p} + \frac{624\pi^2}{425}\, m_b(\alpha_s C_F)^2\, K\,,$$

$$\delta M^{\rm np}_{\Upsilon(1S)} \equiv M_{\Upsilon(1S)} - (2m_b^{\rm PS} + E_1^{\rm p,PS}) \approx [125 \pm 16(\alpha_s) \pm 34(m_b)^{+10}_{-25}(\mu)]\,\text{MeV}\,,$$

$$\begin{aligned} \delta_{\rm np}\Gamma_{\ell\ell}(\Upsilon(1S)) &= \frac{4\alpha^2\alpha_s}{9}\,\frac{17.54 \times 425}{3744}\,\delta M^{\rm np}_{\Upsilon(1S)} \\ &\approx [1.28^{+0.17}_{-0.18}(\alpha_s) \pm 0.42(m_b)^{+0.20}_{-0.57}(\mu) \pm 0.12(m_c)]\,\text{keV}\,. \end{aligned}$$

Conclusions

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- Large NNNLO correction
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- Application: decay of $\Upsilon(1S)$
  for top pair production see talk by M. Steinhauser
- Perturbative corrections well under control
- Non-perturbative corrections sizeable and difficult to quantize